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Connectivity-Preserving Formation Stabilization of Unicycles in Local Coordinates Using Minimum Spanning Tree

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Abstract— **In this paper, we present a novel formation control method to stabilize the positions of a multiagent team moving in a two-dimensional environment to a specified rigid pattern. Agent interactions are typically range-constrained in this kind of system, which makes it critical to maintain the connectivity of the underlying network formed by the mobile agents to enable completion of the desired task. To address this issue, we study the problem of connectivity preservation coupled with the formation control objective. Our contribution is a globally stable formation stabilization approach that maintains connectivity and is designed for unicycle kinematics. Each agent computes its motion using the relative positions of the other agents, expressed in its local arbitrarily oriented coordinate frame. To preserve connectivity, our method relies on a procedure where the desired formation is adaptively scaled to ensure maintenance of the links in the Minimum-distance Spanning Tree of the communications graph. This way, instead of requiring additional control components for connectivity management which may interfere with the formation control objective, we integrate the two goals in the same, bounded, control input. We show formally that the controller provides global stability and ensures connectivity maintenance, and we illustrate its performance in simulation.**

I. Introduction

Multiagent systems can perform complex tasks efficiently and reliably, due to which their application in real-world scenarios is continuously growing. The control of formations [1], [2] is a relevant topic in this field. We address here the problem of formation stabilization, where a team of mobile agents is tasked with achieving a rigid geometric pattern.

It is clearly interesting to solve this problem with each agent using measurements from its independent onboard sensors (e.g., vision). This way, one avoids relying on GPS or other external sensing sources that are not always available (e.g., indoors) or convenient to use. However, the absence of global reference frames in this scenario makes formation control challenging. It induces nonlinear system dynamics, which implies that global stability guarantees are considerably more difficult to obtain than, e.g., with linear consensus-based approaches using common coordinate references [3]. Rigid-formation control methods without global coordinate references typically achieve only local convergence [2], [4] even if the agents use global information. Global stability has been obtained via the use of leader agents [5], [6], dynamic coordinate agreement between the agents [7]–[9], or added time-parameterized control perturbations [10]. A globally stable rigid-formation stabilization method for single integrators using relative position measurements expressed in the agents' independent local frames was proposed in [11], and is at the basis of the work we present. Here, we incorporate connectivity guarantees and we assume unicycle agents, two aspects not considered in that previous controller.

The type of control system we consider is based on message-passing via range-constrained wireless communications. Thus, maintaining the connectivity of the underlying network formed by the agents is critical for successful completion of the desired task. For this reason, connectivity maintenance in multiagent teams has been an abundantly researched problem [12]. Potential field-based methods, which have been often used for the purpose, prevent loss of inter-agent links by introducing additional control terms. These may interfere with the desired control task and can result in unbounded inputs. The latter problem is sorted out by several works that address distributed connectivity-preserving agent consensus [13], [14] and formation stabilization [15]. Some authors have focused on the challenging issue of minimizing the impact of connectivity maintenance on the completion of collective tasks, e.g., deployment [16] or formation control [17]. The methods in [18]–[20] address connectivity management with unicycle agents. We note that, unlike the method we propose, all the works cited in the paragraph require the existence of common coordinate references.

Our contribution is a rigid-formation stabilization approach that uses no global coordinate references, is globally stable, preserves connectivity from any initially connected team configuration, and is designed for unicycle agents. To the best of our knowledge, no existing method has this set of properties. Our key idea is to adaptively scale the desired formation, in a way that incorporates the connectivity maintenance objective without interfering with the formation control objective: the agents' motions, generated by bounded inputs, are always driven by the pursuit of the formation. We maintain connectivity by keeping the links of a spanning tree of the communications graph. Spanning trees are an attractive choice for this purpose, due to their great importance in multiagent networked systems [16]. In particular, we exploit the method presented in [21] where the Minimum-distance Spanning Tree, computed on an event-triggered basis to alleviate communication demands, is used. This specific graph has the interesting property of allowing the agents more leeway to move, as they attain the desired formation, than other possible spanning trees.

In our approach, the agents use global information. Let us note that globally stable stabilization of rigid formations in

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the absence of common frames is a challenging problem even with global information, and that methods for connectivity preservation frequently use global information (e.g., the Laplacian matrix of the current communications graph) [12]. Furthermore, we require no central unit, as each agent can compute independently its control law.

II. Problem formulation

We consider a group of $N > 2$ mobile agents in $\mathbb{R}^2$, define $\mathcal{N}$ as the set of their indexes and denote, in an arbitrary global reference frame, the position of agent $i \in \mathcal{N}$ as $\mathbf{q_i} = [q_i^x, q_i^y]^T \in \mathbb{R}^2$ and its orientation as $\phi_i \in \mathbb{R}$. We assume the motion of each agent i is governed by a unicycle kinematic model (see Fig. 1), as follows:

$$\dot{q}_i^x = -v_i \sin\phi_i, \;\; \dot{q}_i^y = v_i \cos\phi_i, \;\; \dot{\phi}_i = \omega_i, \tag{1}$$

where v_i is its linear velocity and ω_i is its angular velocity. We define a desired configuration, or formation shape, by a certain, fixed, reference layout of the positions of the N agents in their configuration space. The way in which we encode the desired configuration is through a set of interagent relative position vectors. Let us define $\mathbf{q_{ji}} = \mathbf{q_j} - \mathbf{q_i}$, and denote as $\mathbf{c_{ji}} \in \mathbb{R}^2$, $\forall\ i \neq j \in \mathcal{N}$ the vector from i to j in the reference layout of the agents that defines the desired configuration. The agents are not interchangeable, i.e., each of them has a fixed place in the target formation. We then consider that the agents are in the desired configuration if the reference layout has been achieved, up to an arbitrary rotation and translation, i.e., if it holds that:

$$\mathbf{q_{ji}} = \mathbf{R_p c_{ji}}, \;\; \forall i, j \in \mathcal{N}, \tag{2}$$

where $\mathbf{R_p} \in SO(2)$ is a rotation matrix.

Our formation control methodology requires every agent $i \in \mathcal{N}$ to obtain an estimation of the relative position vector to all other agents. For this, i is equipped with communication resources to bidirectionally exchange messages with all agents in a disk of radius R_c centered on $\mathbf{q_i}$. Then, i can obtain the global information through the communications network formed by the group. We define a dynamic undirected communications graph, $\mathcal{G}_c(t) = (\mathcal{N}, \mathcal{E}_c(t))$, for this network, such that every node is associated with an agent, and an edge $\{i, j\}$ exists in $\mathcal{E}_c$ if $||\mathbf{q_{ji}}|| < R_c$. Clearly, if $\mathcal{G}_c$ is connected, exchange of global information is possible through multi-hop transmission. A disconnected $\mathcal{G}_c$, on the other hand, implies that at least one agent has lost the ability to send and receive the information needed for the control task. Then, achieving the desired formation depends critically on $\mathcal{G}_c$ remaining connected during control execution.

Agents must typically be able to also sense the environment or their neighbors. Similarly to $\mathcal{G}_c$, a sensing graph can be defined to establish which pairs of agents are sensorily connected on the basis of their proximity, using a disk model with radius R_s. In a general case, the two graphs may need to stay connected, which can be achieved by ensuring the connectivity of the more restrictive graph. Hence, we use a generic *interaction radius*, $R_{int} = min(R_s, R_c)$ and its associated graph, $\mathcal{G}_{int}(t) = (\mathcal{N}, \mathcal{E}_{int}(t))$. Similarly, we can

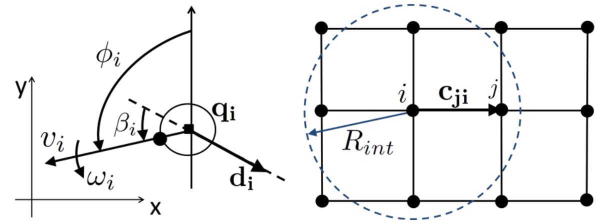

Fig. 1. Left: localization variables and unicycle kinematic model of a given agent i. Desired motion vector $\mathbf{d_i}$ and misalignment angle β_i of the proposed controller (Section III-C). Right: a desired configuration, showing a vector $\mathbf{c_{ji}}$ and the interaction disk, with radius R_{int}.

define $\mathcal{G}_{int}^d = (\mathcal{N}, \mathcal{E}_{int}^d)$ as the static graph of interactions when the agents are in the desired configuration. We assume $\mathcal{G}_{int}^d$ is connected, i.e., $\forall i \in \mathcal{N}$ there exists at least one $j \neq i \in \mathcal{N}$ such that $||\mathbf{c_{ji}}|| < R_{int}$ (Fig. 1). We assume, as well, that a safety factor $0 < f_s < 1$ exists which upper-bounds the distances between neighbors of the desired formation, i.e., $||\mathbf{c_{ji}}|| < f_s{\cdot}R_{int}, \forall \{i, j\} \in \mathcal{E}_{int}^d$. The problem we address is the following:

Problem 1. *Given an initial configuration where the agents are in arbitrary positions such that $\mathcal{G}_{int}$ is connected, define a decentralized control strategy that, using the relative position estimates expressed in the agents' local coordinate frames, achieves the two following objectives:*

- *Maintains the connectivity of $\mathcal{G}_{int}$, allowing every agent to acquire the relative positions of all other agents.*
- *Stabilizes the agents in a set of final positions such that the group is in the desired configuration.*

III. Formation stabilization scheme

Our strategy to stabilize the formation is based on minimizing the following cost function:

$$\gamma = \frac{1}{2N} \sum_{i \in \mathcal{N}} || \sum_{j \in \mathcal{N}} \mathbf{q_{ij}} - \mathbf{R c_{ij}} ||^2, \tag{3}$$

where $\mathbf{R} \in SO(2)$ is a rotation matrix defined in the following section. To show that this function encapsulates the formation control objective, assume $\gamma = 0$ and then consider in (3) the addends associated with two given agents $i = i_1$ and $i = i_2$, which are:

$$\sum_{j \in \mathcal{N}} \mathbf{q_{i_1 j}} - \mathbf{R c_{i_1 j}} = 0, \; \sum_{j \in \mathcal{N}} \mathbf{q_{i_2 j}} - \mathbf{R c_{i_2 j}} = 0. \tag{4}$$

Subtracting the two equations, we have that $\mathbf{q_{i_1 i_2}} = \mathbf{R c_{i_1 i_2}}$, which holds for every pair $i_1, i_2 \in \mathcal{N}$. Hence, the group is in the desired configuration.

A. Rotation matrix

In accordance with the control objective of driving γ to zero, the rotation matrix in (3) is chosen so as to minimize the function, as shown next. We note that the analysis that follows is analogous to solving the orthogonal Procrustes problem [22]. We define $\mathbf{R}$ as a rotation by an angle α, i.e.:

$$\mathbf{R} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}. \tag{5}$$

We can express γ (3) in terms of α and the components of the relative position vectors, $\mathbf{q_{ij}} = [q_{ij}^x, q_{ij}^y]^T$, $\mathbf{c_{ij}} = [c_{ij}^x, c_{ij}^y]^T$:

$$\gamma = \frac{1}{2N}\sum_{i\in\mathcal{N}}\Big[(\sum_{j\in\mathcal{N}} q_{ij}^x - c_{ij}^x\cos\alpha + c_{ij}^y\sin\alpha)^2 \qquad (6)$$
$$+(\sum_{j\in\mathcal{N}} q_{ij}^y - c_{ij}^x\sin\alpha - c_{ij}^y\cos\alpha)^2\Big].$$

Let us define: $\mathbf{S_{qi}} = [S_{qi}^x, S_{qi}^y]^T = \sum_{j\in\mathcal{N}}\mathbf{q_{ij}}$ and $\mathbf{S_{ci}} = [S_{ci}^x, S_{ci}^y]^T = \sum_{j\in\mathcal{N}}\mathbf{c_{ij}}$. To minimize γ with respect to α, we solve $\frac{\partial\gamma}{\partial\alpha} = 0$. After some manipulation, we have:

$$\frac{\partial\gamma}{\partial\alpha} = \frac{1}{N}\Big[\sin\alpha\sum_{i\in\mathcal{N}}(S_{qi}^x S_{ci}^x + S_{qi}^y S_{ci}^y)$$
$$-\cos\alpha\sum_{i\in\mathcal{N}}(-S_{qi}^x S_{ci}^y + S_{qi}^y S_{ci}^x)\Big]. \qquad (7)$$

Then, the condition $\frac{\partial\gamma}{\partial\alpha} = 0$ is expressed as:

$$\sin\alpha\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}} - \cos\alpha\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}^\perp} = 0, \qquad (8)$$

where the superscript $\perp$ denotes a rotation of a vector by $\pi/2$ radians, as follows: $\mathbf{S_{ci}^\perp} = [(0,1)^T, (-1,0)^T]\mathbf{S_{ci}}$. Solving (8) with respect to the rotation angle α gives:

$$\alpha = \arctan\frac{\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}^\perp}}{\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}}}. \qquad (9)$$

Thus, two solutions are possible for α, separated by π radians. To select the correct solution, we compute the second order derivative from (7):

$$\frac{\partial^2\gamma}{\partial\alpha^2} = \frac{1}{N}\Big[\cos\alpha\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}} + \sin\alpha\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}^\perp}\Big]. \qquad (10)$$

From (8) and (10), we can readily see that one of the solutions for (9) minimizes γ, while the other maximizes the function. The solution that is a minimum satisfies the condition $\frac{\partial^2\gamma}{\partial\alpha^2} > 0$. By isolating the term $\cos\alpha$ in (8) and then substituting it in (10), we find that this condition holds when $\sin\alpha/\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}^\perp} > 0$, i.e., $\sin\alpha$ must have the same sign as the numerator of the $\arctan$ function in (9). Hence, among the two possible values of α, the one that minimizes γ, which we will use in our controller, is:

$$\alpha = atan2(\sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}^\perp}, \sum_{i\in\mathcal{N}}\mathbf{S_{qi}^T}\mathbf{S_{ci}}), \qquad (11)$$

where the $atan2$ function returns the solution of (9) for which α is in the appropriate quadrant. The case $atan2(0,0)$, in which α is not defined, is theoretically possible in (11) for degenerate configurations of the agents' positions where γ is constant for all α, see (8). We disregard in our analysis these configurations, which are not attractors within the control strategy we propose, and have measure zero.

We can see now that if the agents are in the desired formation –i.e., if $\mathbf{q_{ji}} = \mathbf{R_p}\mathbf{c_{ji}}$, $\forall i,j\in\mathcal{N}$ for a certain $\mathbf{R_p}$ (2)–, then, clearly, this is the rotation that minimizes γ, and thus $\mathbf{R} = \mathbf{R_p}$ in (3), and $\gamma = 0$. Thus, $\gamma = 0$ if and only if the agents are in the desired configuration, i.e., this condition captures the desired equilibrium of the system.

B. Formation scaling for connectivity preservation

Our formation controller is based on each agent i following the negative gradient of the cost function γ with respect to its position $\mathbf{q_i}$. However, it is easy to see that this strategy can disconnect $\mathcal{G}_{int}$. To maintain connectivity, we propose a method that relies on preserving the links of the Minimum-distance Spanning Tree (MST) of $\mathcal{G}_{int}$. The MST is the spanning tree for which the sum of distances between nodes (agents) is minimum. The use of this tree for managing connectivity in multiagent motion control is interesting because it allows the agents more freedom to move –in order to complete the task (coverage, formation control, flocking...) at hand– than other spanning trees do.

We will keep, in a general case, the links of $\mathcal{G}_m^\mathcal{T} = (\mathcal{N}, \mathcal{E}_m^\mathcal{T})$, a fixed spanning tree of $\mathcal{G}_{int}(t_m)$ defined for a time interval $t\in[t_m, t_{m+1})$ ($m\in\mathbb{N}$ will be used as index for variables defined in this interval). Denote as $\{i_h^m, j_h^m\}$ the edge in $\mathcal{E}_m^\mathcal{T}$ for which the distance between the two agents in the desired formation is maximum. There may be multiple such edges; this is not problematic, since what is relevant to us is simply the value of the maximum distance, as discussed next. The formation controller will drive the pair $\{i_h^m, j_h^m\}$ towards reaching this distance; our goal is to make it reachable without breaking the edge. For this, we use a scaling factor: $0 < S_m \leq 1$, determined at t_m, as follows:

- If $||\mathbf{c_{i_h^m j_h^m}}|| \leq R_{int}$ (the two agents can reach their desired distance) or $\mathcal{G}_{int} \supseteq \mathcal{G}_{int}^d$ (all agents already have all their desired neighbors) we choose $S_m = 1$. In these cases, the values of the distances between agents in the desired formation essentially will not put the maintenance of connectivity at risk.
- If $||\mathbf{c_{i_h^m j_h^m}}|| > R_{int}$, since the desired distance between the two agents is not reachable, we define:

$$S_m = f_s\cdot R_{int}/||\mathbf{c_{i_h^m j_h^m}}||. \qquad (12)$$

We use S_m to redefine, for $t\in[t_m, t_{m+1})$, the desired formation as a possibly down-scaled version of the original one, encoded by the vectors:

$$\mathbf{c_{ij}^m} = S_m\cdot\mathbf{c_{ij}}, \quad \forall i,j\in\mathcal{N}. \qquad (13)$$

The safety margin created by f_s avoids placing the link $\{i_h^m, j_h^m\}$ on the verge of breaking (notice that $|||\mathbf{c^m_{i_h^m j_h^m}}|| = f_s\cdot R_{int}$). Clearly, with the proposed scaling, no link in $\mathcal{G}_m^\mathcal{T}$ will have to be broken to reach the formation. This strategy allows to keep connectivity without additional control terms that may interfere with the formation control goal.

Notice that we can define a cost function in each interval by considering these scaled desired relative position vectors, i.e., $\gamma^m(\mathbf{c_{ij}^m})$, analogous to the function with the desired scale $S_m = 1$, $\gamma(\mathbf{c_{ij}})$. Clearly, $\mathbf{R}$ is not affected by this scaling: from (11), we see that $\mathbf{R^m}(\mathbf{c_{ij}^m}) = \mathbf{R}(\mathbf{c_{ij}})$ and $\alpha^m = \alpha$.

C. Motion control strategy

The control law we define relies on the gradient of the cost function valid at each time instant, which is as follows:

$$\nabla_{\mathbf{q_i}}\gamma^m = \frac{\partial\gamma^m}{\partial\alpha^m}\frac{\partial\alpha^m}{\partial\mathbf{q_i}} + \frac{\partial\gamma^m}{\partial\mathbf{q_i}} = \frac{\partial\gamma^m}{\partial\mathbf{q_i}}, \qquad (14)$$

where we have used that, since the rotation matrix minimizes the cost function (Section III-A), $\frac{\partial \gamma^m}{\partial \alpha^m}$ is null. We want agent i to descend along the gradient (14). After some manipulation, we find the expression for what we call the *desired motion vector*, $\mathbf{d_i^m}$, for agent i:

$$\mathbf{d_i^m} = -\nabla_{\mathbf{q_i}} \gamma^m = \sum_{j \in \mathcal{N}} \mathbf{q_{ji}} - \mathbf{R^m c_{ji}^m}. \tag{15}$$

Hence, to stabilize the formation, each unicycle agent i will follow (within the limits associated with its kinematic constraints) the direction of $\mathbf{d_i^m}$. Still, even having assured, through the formation down-scaling procedure, that all desired distances between agents forming links in $\mathcal{G}_m^{\mathcal{T}}$ are below R_{int}, this does not, in itself, ensure that its longest (i.e., of maximum distance) edge will not exceed R_{int} while our gradient-based control is running. This occurrence must be actively prevented in our method. We define $\mathcal{N}_w = \{i_w, j_w\}$ as the set of two agents forming the current longest edge in $\mathcal{G}_m^{\mathcal{T}}$, and $\mathcal{N}_{nw} = \mathcal{N} - \mathcal{N}_w$. We assume there cannot be multiple longest links at any time. The control law we propose for every agent $i \in \mathcal{N}_{nw}$ is:

$$\begin{cases} v_i = -k_v \, sign(\cos \beta_i) \, ||\mathbf{d_i^m}|| \\ \omega_i = k_\omega \, (\beta_{di} - \beta_i) \end{cases} . \tag{16}$$

All angular quantities we use are expressed in $(-\pi, \pi]$. The angle β_i expresses the misalignment of the agent's heading with respect to its desired motion direction (see Fig. 1), while $k_v > 0$ and $k_\omega > 0$ are control gains, and $\forall i \in \mathcal{N}$:

$$\beta_{di} = \begin{cases} 0 & \text{if} \quad |\beta_i| \leq \frac{\pi}{2} \\ \pi & \text{if} \quad |\beta_i| > \frac{\pi}{2} \end{cases} .$$

We define the predicate: $Pr(i) = (||\mathbf{q_{i_w j_w}}|| \leq f_c R_{int}) \vee \left[\left(\theta_i < \frac{\pi}{2} \wedge \psi_i < \frac{\pi}{2}\right) \vee \left(\left(\frac{\pi}{2} < \theta_i < \pi\right) \wedge \left(\frac{\pi}{2} < \psi_i < \pi\right)\right) \right]$. θ_i is the smallest non-negative angle between $i's$ heading direction and its desired motion vector, while ψ_i is the smallest non-negative angle between the heading direction and the vector from i to the other agent in $\mathcal{N}_w$ (see Fig. 2). f_c is a factor that satisfies $0 < f_s < f_c < 1$ and specifies how close to the limit R_{int} the link preservation procedure starts operating. The control law we propose for $i \in \mathcal{N}_w$ is:

$$\begin{cases} v_i = \begin{cases} -k_v \, sign(\cos \beta_i) \, ||\mathbf{d_i^m}||, & \text{if} \quad Pr(i) \\ 0, \quad \text{otherwise} \end{cases} \\ \omega_i = k_\omega \, (\beta_{di} - \beta_i) \end{cases} . \tag{17}$$

Observe that our agents will travel forwards or backwards, depending on their heading relative to the desired motion vector. In particular, note that an agent's displacement direction is always in the same half-plane as its desired vector. When the longest link in $\mathcal{G}_m^{\mathcal{T}}$ is at risk of breaking (i.e., when $||\mathbf{q_{i_w j_w}}|| > f_c R_{int}$), an agent in that link can rotate like any other agent, but is only allowed to translate (i.e., $Pr(i) = 1$) if the instantaneous displacement generated by the control law would bring it closer to its neighbor in the longest edge. This behavior is captured by the condition in (17) dependent on θ_i and ψ_i. Figure 2 illustrates these aspects.

D. Event-based triggering of MST computation

We select the very interesting option, discussed in Section III-B, of choosing $\mathcal{G}_m^{\mathcal{T}}$ as equal to the MST of $\mathcal{G}_{int}$. However, instead of a continuous update, we use a method presented in [21] to trigger the computation of $\mathcal{G}_m^{\mathcal{T}}$ only when certain events occur. In that work, a new MST was re-calculated only when maintenance of interagent links interfered, by restricting the agents' possible motions, with the execution of the distributed coverage task addressed. Such event-based computation (which is triggered locally by the affected agents) has the important property of reducing network communication demands. In our case, we note an additional interesting characteristic: the event-triggered approach increases the time $(t_{m+1} - t_m)$ between switches of the scale of the desired formation, which contributes to a smoother behavior of our control inputs, with fewer jumps.

Additionally, here we also choose to definitively stop updating $\mathcal{G}_m^{\mathcal{T}}$ and S_m when the event $S_m = 1$ first arises, since the agents can then reach the desired configuration without breaking any of the links of the spanning tree used at that instant. This further reduces communication expenses. Also, we assume the event $\mathcal{G}_{int} \supseteq \mathcal{G}_{int}^d$ can always be detected by the agents, independently from the triggering strategy. Following this event, which makes $S_m = 1$ (Section III-B), we choose as $\mathcal{G}_m^{\mathcal{T}}$ the fixed, preservable MST of $\mathcal{G}_{int}^d$.

E. Reference frames and information requirements

We show in this section that our controller can be computed by each agent in its own arbitrarily oriented coordinate frame, which is a key property, and describe the information it requires the agents to possess. Observe first that the control law for i only requires to know $\mathbf{d_i^m}$ (15), which can be obtained from the desired vectors $\mathbf{c_{ji}}$ and the measurements $\mathbf{q_{ji}}$. Note that the rotation angle α (11) can be computed by each agent from these measurements. Even though we expressed all these variables in a global reference frame (to facilitate the analysis of the controller), i can compute its control law using the vectors expressed in its own, independently oriented local reference frame: $\mathbf{q_{ji}^{L_i}}$. The intuition behind this fact is that the agents move to achieve a formation pattern whose position is centered on the group's centroid, and whose rotation minimizes the sum of squared distances in γ [22], both of which are variables independent of reference frames. We refer to [11] for a formal explanation of these aspects with a cost function analogous to the one used here.

It is easy to see that agent i can integrate in its own reference frame the relative position measurements received from other agents via communications, as long as they share a time reference. Also, i needs to know the other agents' identities, the $\mathcal{G}_m^{\mathcal{T}}$ graph in use and its current longest edge, and the value of the formation scaling factor S_m. We assume that all these data can be obtained by each agent via the spread of global information over the communications network, or through distributed network algorithms.

IV. Stability analysis

Let us first present our result on connectivity maintenance.

Proposition 1. *Under the control strategy proposed and the feedback laws (16) and (17), if $\mathcal{G}_{int}$ is connected at the start of the execution, it will remain connected for all time.*

Proof. The only link in $\mathcal{G}_m^{\mathcal{T}}$ in danger of breaking at any given time t is its longest link: $\{i_w, j_w\}$. Clearly, due to (17), the distances between agents are upper bounded; this implies the control inputs are bounded, and thus the agent positions, and the interagent distances, vary continuously. Then, with the formation down-scaling strategy (Section III-B), if the distance between the two agents forming this link becomes close to R_{int}, they will satisfy $||\mathbf{q_{i_w j_w}}|| \geq f_c R_{int}$ at some instant. By the control law (17), in that situation none of the two agents can perform any displacement that separates it from the other. Therefore, the connectivity of $\mathcal{G}_m^{\mathcal{T}}$ is preserved, and $\mathcal{G}_{int}$ always stays connected. □

The above proposition ensures that our strategy can be implemented: since $\mathcal{G}_{int}$ is connected, every agent can obtain at all times, via multi-hop message-passing, the relative positions of all other agents to compute its control law. Let us now study the stabilization of the system to the desired formation. We start by making the following assumption, which will be used in our subsequent analysis:

Assumption 1. *The length of every interval m, i.e., $(t_{m+1} - t_m)$, is sufficiently large to ensure that, with the control scheme proposed, if $||\mathbf{q_{ij}}(t_m)|| > ||\mathbf{c_{ij}^m}||$, then $||\mathbf{q_{ij}}(t_{m+1})|| < ||\mathbf{q_{ij}}(t_m)||$ $\forall i, j \in \mathcal{N}$.*

Let us explain this Assumption. From (15), it is direct to see that for any two agents $i, j \in \mathcal{N}$ it holds that $(\mathbf{q_i} + \mathbf{d_i^m}/N) - (\mathbf{q_j} + \mathbf{d_j^m}/N) = \mathbf{R c_{ij}^m}$. Thus, since $||\mathbf{R c_{ij}^m}|| = ||\mathbf{c_{ij}^m}||$, if any two agents are currently more distant than they are in the desired configuration, the joint effect of the controller's desired vectors $\mathbf{d}$ is to *always* push them closer. This is illustrated in Fig. 2. It is now clear that Assumption 1 simply implies that, within the limits due to its unicycle kinematics, agent i will be able to follow its $\mathbf{d_i^m}$ closely enough. Note that two agents that are being pushed closer by the desired motion vectors may temporarily separate, due to their unicycle kinematics. Therefore, it is relevant for us to make this assumption. Let us next support its validity. First, it is clear that if we considered agents with single-integrator kinematics, the statement of Assumption 1 would be *automatically* satisfied for any arbitrarily short time interval. The unicycles we actually consider have the time span $(t_{m+1} - t_m)$ to approximate this behavior, and our control law facilitates this: observe that our unicycles can move forwards and backwards (16), (17), which increases their ability to displace, at any instant, in a direction sufficiently close to that of their desired vector $\mathbf{d_i^m}$, when compared with agents that can only move forwards. Also, our angular velocity control is specifically designed to align each agent with the direction of its desired motion vector $\mathbf{d_i^m}$. Qualitatively, the variation of the vectors $\mathbf{d_i^m}$ depends on the variation of the agent positions $\mathbf{q_i}$, which is proportional to the control gain k_v. Then, for a given angular gain k_ω, there will clearly be a small enough k_v such that the resulting variations of the directions of $\mathbf{d_i^m}$ will be slow enough for the agents' headings to follow sufficiently closely. We have observed in simulation that Assumption 1 is comfortably satisfied for usual values of k_v, k_ω.

Proposition 2. *If Assumption 1 holds, the multiagent system with the control strategy proposed and the feedback laws (16) and (17) converges globally to the desired configuration.*

Proof. We will use a Lyapunov-based analysis structured in a number of points that will lead to the stated result:

P1) *Evolution outside of the scale-switching instants*. In every interval $t \in (t_m, t_{m+1})$, we can study the system's behavior by considering a cost function $V^m(t) = \gamma^m(t)$. The dynamics of the function is:

$$\dot{V}^m = \sum_{i \in \mathcal{N}} (\nabla_{\mathbf{q_i}} V^m)^T \dot{\mathbf{q}}_\mathbf{i}. \tag{18}$$

Due to the unicycle kinematics, the variations of the agents positions, $\dot{\mathbf{q}}_\mathbf{i}$, occur always in the direction of their current headings. In addition, the magnitude of the displacement is proportional to the desired motion vector, see (16), (17).

Notice that the angle $\beta_{di} - \beta_i$ expresses the misalignment between the actual translation vector, $\dot{\mathbf{q}}_\mathbf{i}$, and the direction of the desired motion vector $\mathbf{d_i^m}$. Observe, from (15), that (18) captures the dot product of these two vectors. In addition, note that we need to consider separately the terms for the agents forming the longest link in $\mathcal{G}_m^{\mathcal{T}}$. We can thus write:

$$\dot{V}^m = -k_v \Big(\sum_{i \in \mathcal{N}_{nw}} ||\mathbf{d_i^m}||^2 \cos(\beta_{di} - \beta_i) + \sum_{i \in \mathcal{N}_w} Pr(i) ||\mathbf{d_i^m}||^2 \cos(\beta_{di} - \beta_i) \Big). \tag{19}$$

Given that $|\beta_{di} - \beta_i| \leq \pi/2$, it is clear that $\dot{V}^m \leq 0$.

Observe two important properties of our controller. First, from (3) and (15), $\mathbf{d_i^m} = \mathbf{0}, \forall i \in \mathcal{N} \Leftrightarrow \gamma^m = V^m = 0$, i.e., these two equivalent conditions mark the achievement of the scaled version of the desired formation defined in the interval m. Second, from (19), and considering the linear velocities in (16) and (17), any displacement of any agent always reduces V^m. Thus, if $\dot{V}^m = 0$, no agent is displacing. We use these properties in the analysis that follows.

P2) *Stable undesired equilibria are not feasible*. Notice from (19) that it may be possible to have an equilibrium outside of $V^m = 0$ in a situation where $\dot{V}^m = 0$ while, at least for one agent, $\mathbf{d_i^m} \neq \mathbf{0}$ and $\cos(\beta_{di} - \beta_i) = 0$. Let us show next that these equilibria, if they occur, are only temporary (i.e., not stable). If the stated situation holds for any agent in $\mathcal{N}_{nw}$, it will immediately rotate in place, thanks to the angular velocity control (16), and start displacing, thereby lowering V^m. A more careful analysis is needed if the situation holds for an agent in $\mathcal{N}_w$. The two agents in this set behave like any other agent except when $||\mathbf{q_{i_w j_w}}|| > f_c R_{int}$. Assume this situation is satisfied at some instant. Observe that it implies $||\mathbf{q_{i_w j_w}}|| > ||\mathbf{c_{i_w j_w}^m}||$, since the maximum desired distance between agents in any edge of $\mathcal{G}_m^{\mathcal{T}}$ is $f_s R_{int}$. Recalling the reasoning presented right after the statement of Assumption 1, it is then trivial that for at least one of the agents in $\mathcal{N}_w$, the desired motion vector points in a direction that would bring it closer to the other agent. Clearly, in the event of an undesired equilibrium (in which all agent positions are static, as explained above, and thus all $\mathbf{d_i^m}$ (15) are fixed too) this agent will, thanks to the angular control (17), rotate in place to align its heading with its $\mathbf{d_i^m}$. This rotation will always lead to the agent eventually reaching a heading at which it can displace, thereby breaking the equilibrium. As an illustration, see agent j_w in Fig. 2.

Thus, all agents can displace to escape an undesired equilibrium with the possible exception of one of the two agents in $\mathcal{N}_w$. Let us assume that the existence of this non-displacing agent (denote it as i_{nd}) can create an undesired stable equilibrium, where no agent is moving while $V^m > 0$. Since the equilibrium is stable, i.e., it lasts for an arbitrarily long time, this implies that $\mathbf{d_i^m} = \mathbf{0}$ for all agents except i_{nd} (otherwise, if $\mathbf{d_i^m} \neq \mathbf{0}$, any of these agents can rotate in place and then translate, as shown above). Consider two given agents i and j different from i_{nd}. If we substitute in (15), and then subtract, the two equations $\mathbf{d_i^m} = \mathbf{d_j^m} = \mathbf{0}$, we get $\mathbf{q_{ji}} - \mathbf{R^m c_{ji}^m} = \mathbf{0}$. An analogous expression holds for all i, j except i_{nd}. Then, substituting these expressions for agent $i's$ desired vector, $\mathbf{d_i^m} = \mathbf{0}$ (15), we get $\mathbf{q_{i_{nd}i}} - \mathbf{R^m c_{i_{nd}i}^m} = \mathbf{0}$. This implies i and i_{nd} are in the desired configuration with respect to one another. As this reasoning applies to any agent (except i_{nd}) taking the role of i, we have that i_{nd} is in the desired configuration with respect to all agents, i.e., $\mathbf{d_{i_{nd}}^m} = \mathbf{0}$, and $V^m = 0$. We conclude that the only possible stable equilibrium in any given interval m is $V^m = 0$.

P3) *Behavior at the scale-switching instants*. At time t_m, there is a discrete change in the desired formation, as the vectors $\mathbf{c_{ij}^m}$ change discontinuously (13). Thus, the desired vectors $\mathbf{d_i^m}$ (15) change discretely, too. Then, the velocity inputs (16), (17) experience a discontinuous jump. In any case, as the $\mathbf{d_i^m}$ are clearly finite if the agent positions are finite, the values to which the velocity inputs jump are also finite. Hence, the state of the system, in terms of the positions of the agents, does not jump discontinuously.

P4) *S_m converges to unity*. Assume $S_m < 1$ at $t = 0$. All possible temporary desired formations are down-scaled versions of the actual desired one (i.e., $||\mathbf{c_{ij}^m}|| \leq ||\mathbf{c_{ij}}|| \; \forall i, j \in \mathcal{N}, \forall m$). We apply Assumption 1 next. Observe that, regardless of the initial interagent separations and the particular values of S_m, all interagent distances $||\mathbf{q_{ij}}||$ above $||\mathbf{c_{ij}^m}||$ will approach this value as time progresses. Clearly, as long as S_m stays below one, and given that the agent positions evolve continuously (P1, P3) and the system cannot get stuck in undesired equilibria (P2), all pairs of agents are eventually going to satisfy $||\mathbf{q_{ij}}|| \leq ||\mathbf{c_{ij}}||$, which implies $\mathcal{G}_{int} \supseteq \mathcal{G}_{int}^d$, at a certain instant t_{m_f}. As explained in Sections III-B, III-D, this provokes an automatic definitive re-scaling, by virtue of which $S_{m_f} = 1$ and that value is maintained afterwards. S_m may, alternatively, become equal to one earlier, if $||\mathbf{c_{i_h^m j_h^m}}|| \leq R_{int}$, and it will maintain this value subsequently. We stress that, in any possible case, the existence of a final scale switch at a time t_{m_f} is guaranteed.

P5) *Asymptotic stability*. In summary, our control strategy

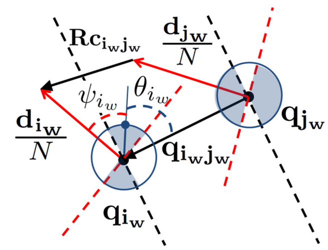

Fig. 2. Motion constraints given by the control strategy when the longest link in $\mathcal{G}_m^{\mathcal{T}}$ is such that $||\mathbf{q_{i_w j_w}}|| > f_c R_{int}$. We omit superindexes, for clarity. These two agents can displace (forwards or backwards) only if their headings lie in the shaded regions, see (17). $\mathbf{d_{j_w}}$ is in the shaded region, whereas $\mathbf{d_{i_w}}$ is not. In our control law, every agent rotates to align its heading with its desired motion vector. Therefore, in a scenario where the two are not displacing, and have arbitrary initial headings, j_w's heading will always end up in the shaded region, but i_w's will not (P2 in Proposition 2).

results in a discontinuous, switched system [23] that we can analyze as follows:

- The agents' positions, which define the state of the system, are always ensured to be finite and change continuously. In addition, the control inputs, (16), (17), stay bounded at all times (P1, P3).
- In $t \in [0, t_{m_f}]$, within every interval m the system monotonically descends along a cost function whose single stable equilibrium encapsulates a down-scaled desired formation (P1, P2).
- For $t > t_{m_f}$, S_m equals one and there are no more scale switches. In this time interval, we can define $V = \gamma$ as a candidate Lyapunov function. This smooth function is clearly radially unbounded, and positive definite, with respect to the desired configuration (Section III-A). Its evolution is such that $\dot{V} \leq 0$ (P1), and the only possible equilibrium is at $V = 0$ (P2). From these dynamic properties, V is clearly a Lyapunov function for the system, *common* across the possible discontinuities in the control laws (16), (17), in the sense of [23]. Therefore, we conclude that the system will converge asymptotically to the desired formation.

$\square$

Our method is based on information acquisition via multi-hop message-passing. Although we do not consider them here, time-delays are an important practical aspect for this type of controller. Formal stability analysis under time-delays of nonlinear interconnected systems such as the one we propose is a challenging issue [24]. In our case, the results in [11] for simpler, single-integrator kinematics can be taken as a reference point to address the study of this problem.

V. Simulation Results

In this section, we evaluate the performance of the proposed multiagent control method in simulation. We present results from an example where a team of 12 agents was tasked with achieving a rectangular grid-shaped formation. Initially, we placed the unicycle agents in positions such that $\mathcal{G}_{int}$ was a path graph. In addition, we chose as initial neighbors in this graph pairs of agents that were to be most distant

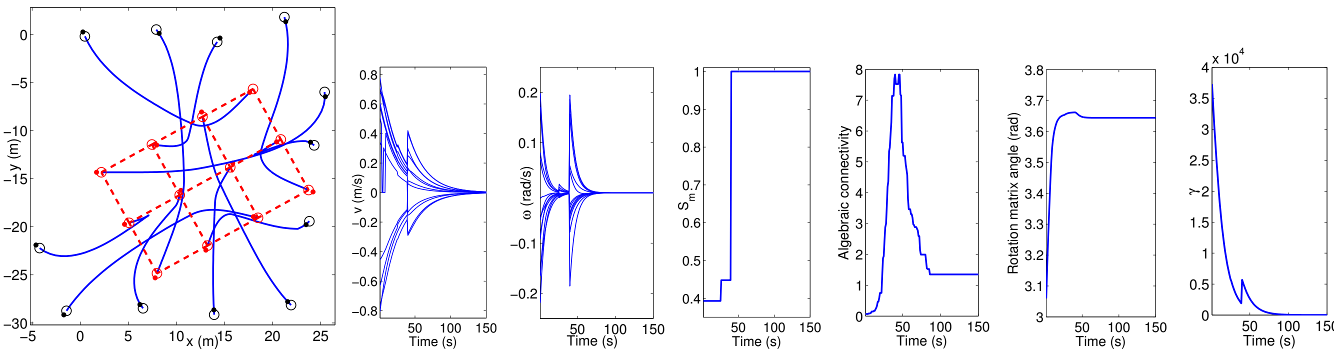

Fig. 3. Simulation results. From left to right: agents' paths (final positions are joined by dashed lines), and evolutions of their linear and angular velocities, the scale factor S_m, the algebraic connectivity, the rotation matrix angle (in a common reference frame), and the piecewise cost functions γ^m.

in the desired configuration, which provides an example of a scenario where connectivity preservation is particularly challenging. The parameters used in the simulation were $R_{int} = 10\ m.$, $f_s = 0.85$, $f_c = 0.9$, while the minimum interagent separation in the desired formation was $6\ m$.

Figure 3 illustrates the results. As can be seen, the agents achieve the specified pattern, with the correct scale. The scale changes of the desired formation as it adapts to the connectivity constraints are depicted. The network remains connected throughout the execution, as shown by the algebraic connectivity (i.e., the second smallest eigenvalue of the Laplacian matrix of $\mathcal{G}_{int}$), which stays above zero –including the very small positive initial value–. The agents move closer in space initially, making connectivity grow. They eventually separate to reach the desired formation and the final connectivity status. The transitions in this process are seamless, as the agents are always *pursuing* the formation.

VI. Conclusion

We have presented a method to stabilize a team of unicycle-type agents in a desired rigid configuration. The approach achieves this goal while preserving connectivity, treating these two objectives in an integrated manner via adaptive scaling of the desired formation. The methodology can be implementable on teams of hundreds of agents with state-of-the-art technology in mobile computing and wireless communications. Future directions of research can include addressing the issue of collision avoidance, which was not considered in this paper and is challenging to study formally, or extending the method to 3D formations or to a partial information-based formation control scenario.

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